ECUACIONES DIFERENCIALES EXACTAS

CRITERIO PARA UNA ECUACIÓN DIFERENCIAL EXACTA

Sean $M(x,y)$ y $N(x,y)$, funciones continuas con derivadas parciales continuas en una región rectangular, R, definida por $a<x<b, c<y<d$. Entonces, la condición necesaria y suficiente para que la ecuación diferencial $M(x,y)dx+N(x,y)dy$ sea exacta es que se cumpla la siguiente identidad $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$

MÉTODO DE SOLUCIÓN EN DERIVE Una vez, que comprobamos que $M(x,y)dx+N(x,y)dy$ es una EDO exacta, activamos el siguiente comando:

EXACT_GEN(M(x,y), N(x,y), x, y, k), para obtener una solución general en términos de constante simbólica K.

Para obtener una solución implícita de la ecuación diferencial exacta, tomando en cuenta las condiciones iniciales se utiliza el comando: **EXACT(M(x,y), N(x,y), x, y, x_0, y_0)**

En caso que la ecuación diferencial no sea identificada por el programa como exacta, enviará el mensaje **"INAPLICABLE".**

EJEMPLO 1. Resuelva la ecuación diferencial $y'=\frac{(xy^2-1)}{(1-x^2y)}$

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#1:   LOAD(C:\Program Files\TI Education\Derive 6 - Evaluación\Math\FirstOrderODEs.mth)
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```
#2:   InputMode := Word
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$$\#3: \quad \frac{dy}{dx}=\frac{x\cdot y^2-1}{1-x^2\cdot y}$$

$$\#4: \quad (x\cdot y^2-1)\cdot dx+(x^2\cdot y-1)\cdot dy=0$$

Identificar: $M(x,y)=xy^2-1$, $N(x,y)=x^2y-1$

```
#5:   InputMode := Character
```

$$\#6: \quad M(x, y) := x\cdot y^2-1$$

$$\#7: \quad \frac{d}{dy}(x\cdot y^2-1)$$

$$\#8: \quad 2\cdot x\cdot y$$

$$\#9: \quad N(x, y) := x^2\cdot y-1$$

$$\#10: \quad \frac{d}{dx}(x^2\cdot y-1)$$

$$\#11: \quad 2\cdot x\cdot y$$

SE COMPRUEBA QUE ES EXACTA

$$\#12: \quad EXACT\_GEN(x\cdot y^2-1,\ x^2\cdot y-1,\ x,\ y,\ k)$$

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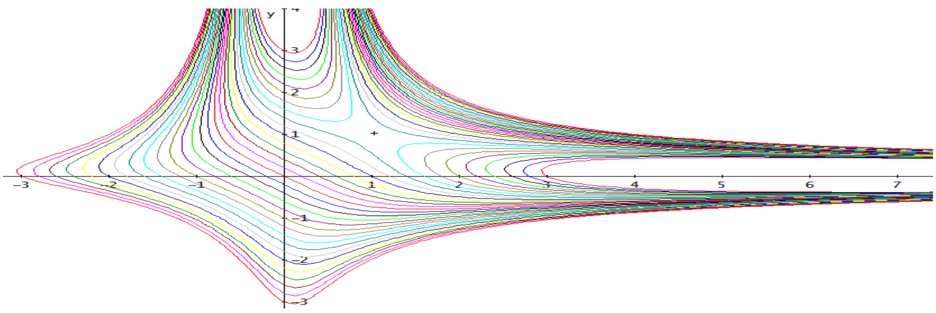
SE COMPRUEBA QUE ES EXACTA

#12: EXACT_GEN$(x\cdot y^2 - 1,\ x^2\cdot y - 1,\ x,\ y,\ k)$

#13:
$$\frac{x^2\cdot y^2}{2} - x - y = k$$

#14:
$$\text{VECTOR}\left(\frac{x^2\cdot y^2}{2} - x - y = k,\ k,\ -3,\ 3,\ 0.2\right)$$

#15:
$$\left[\frac{x^2\cdot y^2}{2} - x - y = -3,\ \frac{x^2\cdot y^2}{2} - x - y = -\frac{14}{5},\ \frac{x^2\cdot y^2}{2} - x - y = -\frac{13}{5},\ \frac{x^2\cdot y^2}{2} - x - y = -\frac{12}{5},\ \frac{x^2\cdot y^2}{2} - x - y = -\frac{11}{5},\ \frac{x^2\cdot y^2}{2} - x - y = -2,\right.$$
$$\frac{x^2\cdot y^2}{2} - x - y = -\frac{9}{5},\ \frac{x^2\cdot y^2}{2} - x - y = -\frac{8}{5},\ \frac{x^2\cdot y^2}{2} - x - y = -\frac{7}{5},\ \frac{x^2\cdot y^2}{2} - x - y = -\frac{6}{5},\ \frac{x^2\cdot y^2}{2} - x - y = -1,\ \frac{x^2\cdot y^2}{2} - x - y = -\frac{4}{5},$$
$$\frac{x^2\cdot y^2}{2} - x - y = -\frac{3}{5},\ \frac{x^2\cdot y^2}{2} - x - y = -\frac{2}{5},\ \frac{x^2\cdot y^2}{2} - x - y = -\frac{1}{5},\ \frac{x^2\cdot y^2}{2} - x - y = 0,\ \frac{x^2\cdot y^2}{2} - x - y = \frac{1}{5},\ \frac{x^2\cdot y^2}{2} - x - y = \frac{2}{5},$$
$$\frac{x^2\cdot y^2}{2} - x - y = \frac{3}{5},\ \frac{x^2\cdot y^2}{2} - x - y = \frac{4}{5},\ \frac{x^2\cdot y^2}{2} - x - y = 1,\ \frac{x^2\cdot y^2}{2} - x - y = \frac{6}{5},\ \frac{x^2\cdot y^2}{2} - x - y = \frac{7}{5},\ \frac{x^2\cdot y^2}{2} - x - y = \frac{8}{5},\ \frac{x^2\cdot y^2}{2} -$$
$$x - y = \frac{9}{5},\ \frac{x^2\cdot y^2}{2} - x - y = 2,\ \frac{x^2\cdot y^2}{2} - x - y = \frac{11}{5},\ \frac{x^2\cdot y^2}{2} - x - y = \frac{12}{5},\ \frac{x^2\cdot y^2}{2} - x - y = \frac{13}{5},\ \frac{x^2\cdot y^2}{2} - x - y = \frac{14}{5},\ \frac{x^2\cdot y^2}{2} - x - y$$
$$\left. = 3\right]$$



EJEMPLO 2. Resuelva la ecuación diferencial $2xy + \left(x^2+1\right)y' = 0$

#1: LOAD(C:\Program Files\TI Education\Derive 6 – Evaluación\Math\FirstOrderODEs.mth)

#2: InputMode := Word

#3: $2\cdot xy + (x^2 + 1)\cdot\frac{dy}{dx} = 0$

#4: $2\cdot xy\cdot dx + (x^2 + 1)\cdot dy = 0$

#5: InputMode := Character

#6: $M(x,\ y) := 2\cdot x\cdot y$

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#7: $\frac{d}{dy}(2\cdot x\cdot y)$

#8: $2\cdot x$

#9: $N(x, y) := x^2 + 1$

#10: $\frac{d}{dx}(x^2 + 1)$

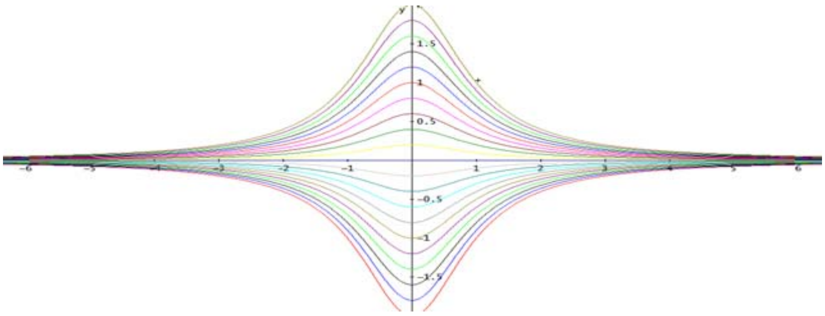
#11: $2\cdot x$

#12: EXACT_GEN($2\cdot x\cdot y$, $x^2 + 1$, x, y, k)

#13: $x^2\cdot y + y = k$

#14: VECTOR($x^2\cdot y + y = k$, k, -2, 2, 0.2)

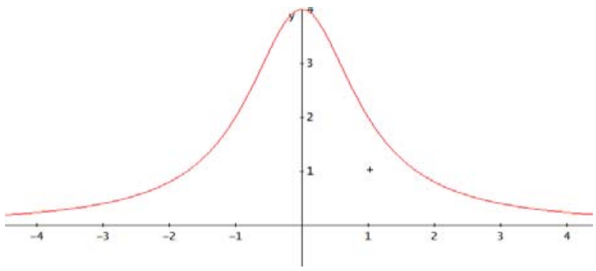
#15: $\left[x^2\cdot y + y = -2,\ x^2\cdot y + y = -\frac{9}{5},\ x^2\cdot y + y = -\frac{8}{5},\ x^2\cdot y + y = -\frac{7}{5},\ x^2\cdot y + y = -\frac{6}{5},\ x^2\cdot y + y = -1,\ x^2\cdot y + y = -\frac{4}{5},\ x^2\cdot y + y = -\frac{3}{5},\ x^2\cdot y + y = -\frac{2}{5},\ x^2\cdot y + y = -\frac{1}{5},\ x^2\cdot y + y = 0,\ x^2\cdot y + y = \frac{1}{5},\ x^2\cdot y + y = \frac{2}{5},\ x^2\cdot y + y = \frac{3}{5},\ x^2\cdot y + y = \frac{4}{5},\ x^2\cdot y + y = 1,\ x^2\cdot y + y = \frac{6}{5},\ x^2\cdot y + y = \frac{7}{5},\ x^2\cdot y + y = \frac{8}{5},\ x^2\cdot y + y = \frac{9}{5},\ x^2\cdot y + y = 2\right]$



Si suponemos que la curva pasa el punto (1.2), determine la solución particular.

#16: EXACT($2\cdot x\cdot y$, $x^2 + 1$, x, y, 1, 2)

#17: $x^2\cdot y + y = 4$



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EJEMPLO 3. Resolver la EDO $\left(e^{2y}-y\ \cos xy\right)dx+\left(2xe^{2y}-x\cos xy+2y\right)dy=0.$

#1: LOAD(C:\Program Files\TI Education\Derive 6 - Evaluación\Math\FirstOrderODEs.mth)

#2: InputMode := Word

#3: $(e^{2\cdot y} - y\cdot \text{COS}(xy))\cdot dx + (2\cdot x\cdot e^{2\cdot y} - x\cdot \text{COS}(xy) + 2\cdot y)\cdot dy = 0$

#4: InputMode := Character

#5: $M(x,\ y) := e^{2\cdot y} - y\cdot \text{COS}(x\cdot y)$

#6: $\frac{d}{dy}(e^{2\cdot y} - y\cdot \text{COS}(x\cdot y))$

#7: $2\cdot e^{2\cdot y} - \text{COS}(x\cdot y) + x\cdot y\cdot \text{SIN}(x\cdot y)$

#8: $N(x,\ y) := 2\cdot x\cdot e^{2\cdot y} - x\cdot \text{COS}(x\cdot y) + 2\cdot y$

#9: $\frac{d}{dx}(2\cdot x\cdot e^{2\cdot y} - x\cdot \text{COS}(x\cdot y) + 2\cdot y)$

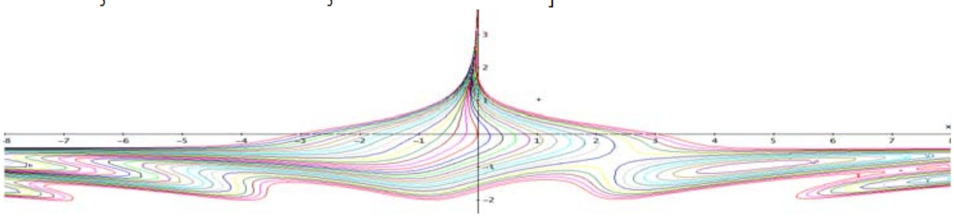
#10: $2\cdot e^{2\cdot y} - \text{COS}(x\cdot y) + x\cdot y\cdot \text{SIN}(x\cdot y)$

#11: $\text{EXACT\_GEN}(e^{2\cdot y} - y\cdot \text{COS}(x\cdot y),\ 2\cdot x\cdot e^{2\cdot y} - x\cdot \text{COS}(x\cdot y) + 2\cdot y,\ x,\ y,\ k)$

#12: $x\cdot e^{2\cdot y} - \text{SIN}(x\cdot y) + y^2 = k$

#13: $\text{VECTOR}(x\cdot e^{2\cdot y} - \text{SIN}(x\cdot y) + y^2 = k,\ k,\ -3,\ 3,\ 0.2)$

#14: $\left[x\cdot e^{2\cdot y} - \text{SIN}(x\cdot y) + y^2 = -3,\ x\cdot e^{2\cdot y} - \text{SIN}(x\cdot y) + y^2 = -\frac{14}{5},\ x\cdot e^{2\cdot y} - \text{SIN}(x\cdot y) + y^2 = -\frac{13}{5},\ x\cdot e^{2\cdot y} - \text{SIN}(x\cdot y) + y^2 = -\frac{12}{5},\ x\cdot e^{2\cdot y} - \text{SIN}(x\cdot y) + y^2 = -\frac{11}{5},\ x\cdot e^{2\cdot y} - \text{SIN}(x\cdot y) + y^2 = -2,\ x\cdot e^{2\cdot y} - \text{SIN}(x\cdot y) + y^2 = -\frac{9}{5},\ x\cdot e^{2\cdot y} - \text{SIN}(x\cdot y) + y^2 = -\frac{8}{5},\ x\cdot e^{2\cdot y} - \text{SIN}(x\cdot y) + y^2 = -\frac{7}{5},\ x\cdot e^{2\cdot y} - \text{SIN}(x\cdot y) + y^2 = -\frac{6}{5},\ x\cdot e^{2\cdot y} - \text{SIN}(x\cdot y) + y^2 = -1,\ x\cdot e^{2\cdot y} - \text{SIN}(x\cdot y) + y^2 = -\frac{4}{5},\ x\cdot e^{2\cdot y} - \text{SIN}(x\cdot y) + y^2 = -\frac{3}{5},\ x\cdot e^{2\cdot y} - \text{SIN}(x\cdot y) + y^2 = -\frac{2}{5},\ x\cdot e^{2\cdot y} - \text{SIN}(x\cdot y) + y^2 = -\frac{1}{5},\ x\cdot e^{2\cdot y} - \text{SIN}(x\cdot y) + y^2 = 0,\ x\cdot e^{2\cdot y} - \text{SIN}(x\cdot y) + y^2 = \frac{1}{5},\ x\cdot e^{2\cdot y} - \text{SIN}(x\cdot y) + y^2 = \frac{2}{5},\ x\cdot e^{2\cdot y} - \text{SIN}(x\cdot y) + y^2 = \frac{3}{5},\ x\cdot e^{2\cdot y} - \text{SIN}(x\cdot y) + y^2 = \frac{4}{5},\ x\cdot e^{2\cdot y} - \text{SIN}(x\cdot y) + y^2 = 1,\ x\cdot e^{2\cdot y} - \text{SIN}(x\cdot y) + y^2 = \frac{6}{5},\ x\cdot e^{2\cdot y} - \text{SIN}(x\cdot y) + y^2 = \frac{7}{5},\ x\cdot e^{2\cdot y} - \text{SIN}(x\cdot y) + y^2 = \frac{8}{5},\ x\cdot e^{2\cdot y} - \text{SIN}(x\cdot y) + y^2 = \frac{9}{5},\ x\cdot e^{2\cdot y} - \text{SIN}(x\cdot y) + y^2 = 2,\ x\cdot e^{2\cdot y} - \text{SIN}(x\cdot y) + y^2 = \frac{11}{5},\ x\cdot e^{2\cdot y} - \text{SIN}(x\cdot y) + y^2 = \frac{12}{5},\ x\cdot e^{2\cdot y} - \text{SIN}(x\cdot y) + y^2 = \frac{13}{5},\ x\cdot e^{2\cdot y} - \text{SIN}(x\cdot y) + y^2 = \frac{14}{5},\ x\cdot e^{2\cdot y} - \text{SIN}(x\cdot y) + y^2 = 3\right]$



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EJEMPLO 4. Resolver la ecuación diferencial $\dfrac{dy}{dx}=\dfrac{x-y\cos x}{senx+y}$

#1: LOAD(C:\Program Files\TI Education\Derive 6 - Evaluación\Math\FirstOrderODEs.mth)

#2: InputMode := Word

#3: $\dfrac{dy}{dx}=\dfrac{x - ycos\cdot x}{SIN(x) + y}$

#4: $\dfrac{dy}{dx}=\dfrac{x - y\cdot COS(x)}{SIN(x) + y}$

#5: $(y\cdot COS(x) - x)\cdot dx + (SIN(x) + y)\cdot dy = 0$

#6: InputMode := Character

#7: $M(x, y) := y\cdot COS(x) - x$

#8: $\dfrac{d}{dy}(y\cdot COS(x) - x)$

#9: $COS(x)$

#10: $N(x, y) := SIN(x) + y$

#11: $\dfrac{d}{dx}(SIN(x) + y)$

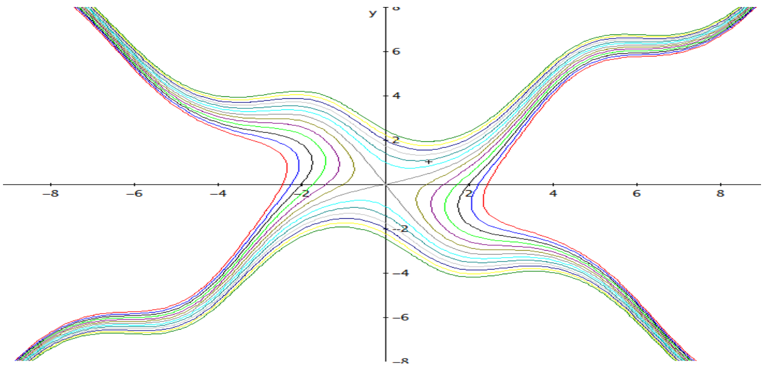
#12: $COS(x)$

#13: EXACT_GEN(y·COS(x) - x, SIN(x) + y, x, y, k)

#14: $y\cdot SIN(x) - \dfrac{x^2}{2} + \dfrac{y^2}{2} = k$

#15: $VECTOR\left(y\cdot SIN(x) - \dfrac{x^2}{2} + \dfrac{y^2}{2} = k, k, -3, 3, 0.5\right)$

#16: $\left[y\cdot SIN(x) - \dfrac{x^2}{2} + \dfrac{y^2}{2} = -3, y\cdot SIN(x) - \dfrac{x^2}{2} + \dfrac{y^2}{2} = -\dfrac{5}{2}, y\cdot SIN(x) - \dfrac{x^2}{2} + \dfrac{y^2}{2} = -2, y\cdot SIN(x) - \dfrac{x^2}{2} + \dfrac{y^2}{2} = -\dfrac{3}{2}, y\cdot SIN(x) - \dfrac{x^2}{2} + \dfrac{y^2}{2} = -1, y\cdot SIN(x) - \dfrac{x^2}{2} + \dfrac{y^2}{2} = -\dfrac{1}{2}, y\cdot SIN(x) - \dfrac{x^2}{2} + \dfrac{y^2}{2} = 0, y\cdot SIN(x) - \dfrac{x^2}{2} + \dfrac{y^2}{2} = \dfrac{1}{2}, y\cdot SIN(x) - \dfrac{x^2}{2} + \dfrac{y^2}{2} = 1, y\cdot SIN(x) - \dfrac{x^2}{2} + \dfrac{y^2}{2} = \dfrac{3}{2}, y\cdot SIN(x) - \dfrac{x^2}{2} + \dfrac{y^2}{2} = 2, y\cdot SIN(x) - \dfrac{x^2}{2} + \dfrac{y^2}{2} = \dfrac{5}{2}, y\cdot SIN(x) - \dfrac{x^2}{2} + \dfrac{y^2}{2} = 3\right]$



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EJEMPLO 5. Resolver: $\cos x dy - \left(2 ysenx - 3\right) dx = 0$

```
#1:   LOAD(C:\Program Files\TI Education\Derive 6 - Evaluación\Math\FirstOrderODEs.mth)

#2:   InputMode := Word

#3:   COS(x)·dy - (2·y·SIN(x) - 3)·dx = 0

#4:   InputMode := Character

#5:   M(x, y) := 3 - 2·y·SIN(x)

      d
#6:   ── (3 - 2·y·SIN(x))
      dy

#7:                                                              - 2·SIN(x)

#8:   N(x, y) := COS(x)

      d
#9:   ── COS(x)
      dx

#10:                                                               - SIN(x)

#11:  EXACT_GEN(3 - 2·y·SIN(x), COS(x), x, y, k)

#12:                                                            inapplicable
```

Por lo tanto no es una EDO exacta

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